J. Appl. Maths Mechs Vol. 57, No. 3, pp. 503–510, 1993
Printed in Great Britain.

0021–8928/93 $24.00+.00

THE SELF-SIMILAR PROBLEM OF THE ACTION OF A MOVING LOAD AT THE BOUNDARY OF A NON-LINEAR ELASTIC WEAKLY ANISOTROPIC HALF-SPACE†

A. P. CHUGAINOVA

Crimea Region

(*Received* 22 *May* 1991)

The solution of the self-similar problem of the action of a moving load at the boundary of a non-linear elastic weakly anisotropic half-space is investigated. A solution is constructed in the form of a system of quasi-longitudinal and quasi-transverse two-dimensional stationary simple and shock waves. Non-linear effects turn out to be significant when constructing the quasi-transverse wave system.

THIS PAPER is a direct continuation of [1], which investigated stationary two-dimensional simple and shock waves. The problem of the effect of non-linearity on the reflection of a weak longitudinal shock wave from a plane boundary in an elastic medium was discussed in [2, 3], and it was shown that the non-linearity has a significant effect on the solution if the angle between the wave front and the wall is close to the limiting one (in the case of an isotropic body, close to $\pi/2$). It is shown below that, in the general case, a small non-linearity and anisotropy governs at the principal order the nature and sequence of the quasi-transverse waves. When the non-linearity and anisotropy tend to zero, the domain occupied by these waves decreases and in the limit they merge into a single transverse wave.

Solutions have been given [4] for particular plane self-similar boundary-value problems of the reflection of shock waves from the boundary of an isotropic non-linear elastic half-space.

1. Suppose that a non-linear elastic weakly anisotropic non-heat-conducting medium occupies the half-space $\eta_1 \geq 0$ (η_i, $i = 1, 2, 3$ are Lagrangian coordinates and are rectangular Cartesian coordinates in the unstressed state), and that stresses are applied to the $\eta_1 = 0$ plane taking constant values in each of the half-planes $\eta_1 = 0$, $\eta_2 < Wt$ and $\eta_1 = 0$, $\eta_2 > Wt$ ($W = \text{const}$).

We will introduce a system of coordinates ν_1, ν_2, ν_3: $\nu_1 = \eta_1$, $\nu_2 = \eta_2 - Wt$, $\nu_3 = \eta_3$ in which the problem is self-similar: the solution depends on ν_1/ν_2. If W is sufficiently large [1–3], the perturbation from the boundary $\nu_1 = 0$ into the domain $\nu_1 > 0$ propagates in the form of two-dimensional simple and shock waves.

We shall assume that the stresses $\Delta\sigma_{ij}$ which appear in the medium during the passage of the wave are small and that their order does not exceed ε. One can then consider the problem of the action of the moving load within a linearized framework. As a first approximation we will take the solution of the linear problem for an isotropic medium. The solution of the non-linear problem for a weakly anisotropic medium will be found as the next approximation.

The change in the stress tensor can be represented in the form of a sum $\Delta\sigma = \Delta_1\sigma + \Delta_2\sigma$, where $\Delta_1\sigma$ is the change of the stresses in the longitudinal wave, and $\Delta_2\sigma$ is the change in the

†*Prikl. Mat. Mekh.* Vol. 57, No. 3, pp. 102–111, 1993.

transverse wave. For an explicit calculation of the latter quantities we introduce two systems of coordinates ζ_i' and ζ_i'' obtained from system v_i by a rotation about the v_3 axis through angles $\alpha = \arcsin\sqrt{(\mu/(\rho_0 W^2))}$ and $\alpha_1 = \arcsin\sqrt{((\lambda+2\mu)/(\rho_0 W^2))}$, so that the ξ_2' and ξ_2'' axes are directed along the fronts of the corresponding waves. The components of the stress tensor in the ξ_i' and ξ_i'' coordinates are denoted by σ_{ij}' and σ_{ij}''. Then $\Delta\sigma_{11}''$ is the only non-zero component in the longitudinal wave in the tensor $\Delta_1\sigma$, and $\Delta\sigma_{21}'$ and $\Delta\sigma_{31}'$ are the only non-zero components in the transverse wave in the tensor $\Delta_2\sigma$. Returning to system v_i and using the boundary conditions, we obtain

$$\Delta\sigma_{11} = \cos^2\alpha_1 \Delta\sigma_{11}'' + \sin\alpha\cos\alpha\,\Delta\sigma_{21}' = \Delta\sigma_{11}^\gamma$$

$$\Delta_{21} = -\sin\alpha_1\cos\alpha_1\,\Delta\sigma_{11}'' + \cos^2\alpha\,\Delta\sigma_{21}' = \Delta\sigma_{21}^\gamma$$

$$\Delta_{31} = \cos\alpha\,\Delta\sigma_{31}' = \Delta\sigma_{31}^\gamma$$

where $\Delta\sigma_{i1}^\gamma$ are the stress components specified at the boundary.

These relations enable us to express $\Delta\sigma_{11}''$, $\Delta\sigma_{21}'$ and $\Delta\sigma_{31}'$ in terms of the boundary stresses, i.e. to determine the intensity of the waves occurring in the solution.

Under the variation $\Delta\sigma_{11}''$ the point σ_{11}, σ_{21}, σ_{31} in stress space moves along a straight line, and under the variation $\Delta\sigma_{21}'$, $\Delta\sigma_{31}'$ this point describes a plane. The angle β between the line and the normal to the plane is given by the formula $\cos\beta = \cos(\alpha_1 - \alpha)$.

The angle $\beta = 0$ only if $\alpha_1 = \alpha$. The difference $\alpha_1 - \alpha$ is always finite, because the angle between the line and the plane cannot equal $\pi/2$. The angle between the line and the plane cannot vanish because $\beta = \pi/2$ only when $\alpha_1 = \alpha + \pi/2$, which is impossible.

The above-mentioned line in the problem under consideration should pass through a point corresponding to the initial value of the stress σ_{i1}^0 (before the passage of the wave), and the plane through the point corresponding to the variable stresses σ_{i1}^γ ($\sigma_{i1}^\gamma = \sigma_{i1}^0 + \Delta\sigma_{i1}^\gamma$). The point of intersection A between the line and the plane enables one to determine the stresses σ_{i1}^* in the domain between the longitudinal and transverse waves.

When one considers non-linear waves, a curve L, corresponding to quasi-longitudinal waves, passes through the initial state. The curve L is a segment of the integral curve of a simple non-reversing wave or an evolutionary segment of a shock polar [5]. For simplicity one can consider the projection of the curve L, specified in the nine-dimensional σ_{ij} space, onto the three-dimensional σ_{i1} space. At any point A on the curve L one can construct a two-dimensional surface S_A, corresponding to the variation of quantities in a succession of two quasi-transverse waves [1], which we shall also project onto the three-dimensional subspace σ_{i1}. To solve the non-linear problem it is necessary to choose the point A such that the surface passes through the point σ_{i1}^γ, in accordance with the boundary conditions. Then the coordinates of the point A specify the stresses between the quasi-longitudinal and the first of the quasi-transverse waves. Because the curve L and the surfaces S_A have tangents [1, 5], it is obvious that if the points σ_{i1}^0 and σ_{i1}^γ are sufficiently close, the problem becomes linear and, consequently, has a unique solution.

The weak non-linearity and anisotropy cause the transverse wave to decompose into two or more quasi-transverse waves, moving with nearly identical velocities. Ignoring details associated with the decomposition of the transverse wave to a first approximation into two or more quasi-transverse simple and shock waves, the solution of the weakly non-linear problem with small anisotropy is close to the solution in the first approximation.

In particular, one can show that the position of the point A differs from its position in the first approximation by a quantity of order $\chi = \max\{\varepsilon^2, g\varepsilon\}$, where g is the anisotropy parameter. This is connected with the fact that in the domain under consideration, with size of order ε, the angle between the tangent to the curve L and the corresponding line in the first approximation, and also between the tangent planes to S_A and the corresponding plane in the first approximation, have an order of magnitude no greater than $\{\varepsilon, g\}$.

The effects on the solution of non-linearity, together with the strong influence of small anisotropy, appear in quasi-transverse wave behaviour in the same way as in one-dimensional non-stationary problems [6, 7]. The behaviour of quantities in quasi-transverse stationary two-dimensional simple and shock waves is the same as the behaviour of one-dimensional non-stationary simple and shock waves [1] up to an accuracy of order χ, and, consequently, these waves can be constructed in the same way as in the one-dimensional non-stationary case. However, unlike the one-dimensional non-stationary case, here the quasi-transverse waves propagate through a medium whose state is not specified in advance, but is "prepared" by the quasi-longitudinal wave.

2. To describe quasi-transverse wave processes in a non-linear elastic medium with a small anisotropy of general form, we will introduce a two-dimensional potential F [8] (as a function of σ_{21} and σ_{31}) which occurs in the equation of motion of the two-dimensional stationary quasi-transverse waves and jump conditions

$$F = \tfrac{1}{2}(f-g)\sigma_{31}'^2 + \tfrac{1}{2}\sigma_{21}'^2 - \tfrac{1}{8}\kappa(\sigma_{31}'^2 + \sigma_{21}'^2)^2 +$$
$$+ s\sigma_{21}'\sigma_{31}' + \tfrac{1}{2}(p\sigma_{21}' + q\sigma_{31}')(\sigma_{31}'^2 + \sigma_{21}'^2) + e\sigma_{31}'^3 + d\sigma_{21}'^3$$

where f, g, κ, s, p, q, e and d are constants for which expressions were obtained in [8]. The constants f, g and κ have the following physical interpretations: the small quantity g is the anisotropy parameter, p, q, e, $d \sim g$, f/μ is the characteristic velocity when there is no non-linearity or anisotropy, and κ is the elastic constant of the medium and characterizes the non-linear properties of the medium in quasi-transverse waves.

When $d=e=0$, e.g. for media with some symmetry (such as a transversely isotropic or orthotropic elastic medium), by translating the coordinate axes in the σ_{21}', σ_{31}' plane one can remove cubic terms in the expression for F [8]. The new origin of coordinates is found at the point $O_*(2q/\kappa;\ 2p/\kappa)$. By subsequent rotation about the new origin of coordinates O_* through an angle φ the function F for the anisotropic medium is reduced to the form

$$F = \tfrac{1}{2}(f^n - g^n)\sigma_{31}^{n2} + \tfrac{1}{2}(f^n + g^n)\sigma_{21}^{n2} - \tfrac{1}{8}\kappa(\sigma_{31}^{n2} + \sigma_{21}^{n2})^2$$

$$f^n = f + 4\frac{p^2+q^2}{\kappa};\quad g^n = \left[\left(g + 2\frac{p^2-q^2}{\kappa}\right)^2 + \left(s + 4\frac{pq}{\kappa}\right)^2\right]^{1/2}$$

$$\begin{aligned} \sigma_{21}^n &= (-\sigma_{31}' + 2q/\kappa)\sin\varphi + (\sigma_{21}' - 2p/\kappa)\cos\varphi \\ \sigma_{31}^n &= (\sigma_{31}' - 2q/\kappa)\cos\varphi + (\sigma_{21}' - 2p/\kappa)\sin\varphi \\ \operatorname{tg}2\varphi &= -(s + 4pq/\kappa)[g + 2(p^2 - q^2)/\kappa] \end{aligned} \tag{2.1}$$

Below we shall investigate quasi-transverse waves in axes σ_{21}^n, σ_{31}^n with the superscript n omitted.

Two quasi-transverse simple waves participate in the solution of the non-linear problem. The angles between the ν_2 axis and the direction of propagation of these two waves is governed by the relations [1]

$$\begin{aligned} &\theta_{1,2} = \alpha + \psi_{1,2}, \quad \alpha = \arcsin\sqrt{\mu/\rho_0 W^2} \\ &\psi_{1,2} = A^{-2}[C^2 - f + \kappa(\sigma_{21}^2 + \sigma_{31}^2 \pm ((\sigma_{31}^2 - \sigma_{21}^2 + 2q/\kappa)^2 + 4\sigma_{31}^2\sigma_{21}^2)^{1/2})] \\ &A^2 = 2\rho_0 W^2 \sin\alpha\cos\alpha, \quad C^2 = \rho_0 W^2 \sin^2\alpha \end{aligned} \tag{2.2}$$

The properties of quasi-transverse waves were investigated in [1] using variables $l_i = \partial w_i / \partial \nu_i$, where the w_i are the components of the displacement vector. The following relations hold between the variables l_i $(i=2, 3)$ and σ_{21}, σ_{31}

$$\sigma_{21}=\mu l_2+O(\varepsilon\chi), \quad \sigma_{31}=\mu l_3+O(\varepsilon\chi) \tag{2.3}$$

The integral curves of stationary quasi-transverse simple waves are described by the differential equations [1]

$$\frac{d\sigma_{21}}{d\sigma_{31}}=\frac{\sigma_{21}^2-\sigma_{31}^2-2g/\kappa\mp((\sigma_{21}^2-\sigma_{31}^2-2g/\kappa)^2+4\sigma_{21}^2\sigma_{31}^2)^{1/2}}{2\sigma_{21}\sigma_{31}} \tag{2.4}$$

The form of the integral curves (2.4) and the variation of the quantities $\psi_{2,1}$ along them was investigated in [1].

The shock polar of stationary quasi-transverse shock waves in the σ_{21}, σ_{31} plane is given by the equation [1]

$$\begin{gathered}(\sigma_{21}^2+\sigma_{31}^2-R^2)(\overset{*}{\sigma}_{31}\sigma_{21}-\overset{*}{\sigma}_{21}\sigma_{31})+2g\kappa^{-1}(\overset{*}{\sigma}_{21}-\sigma_{21})(\overset{*}{\sigma}_{31}-\sigma_{31})=0\\ R^2=\overset{*}{\sigma}{}_{21}^2+\overset{*}{\sigma}{}_{31}^2\end{gathered} \tag{2.5}$$

Here σ_{i1}^* $(i=2, 3)$ is the stress in front of the quasi-transverse shock wave. The form of the shock polar (2.5) and its segments, which simultaneously satisfy the non-decreasing entropy and evolution conditions, are given in [1].

To construct the part of the solution of the moving load problem that corresponds to the quasi-transverse wave system, it is necessary in the σ_{21}, σ_{31} plane to connect the point σ_{i1}^* corresponding to the state behind the quasi-longitudinal wave to the point σ_{i1}^γ corresponding to the boundary conditions $(\sigma_{i1}^\gamma=\sigma_{i1}^0+\Delta\sigma_{i1}^\gamma)$, using the integral curves of the quasi-transverse simple non-reversing waves and the evolving segments of shock polars of quasi-transverse shock waves, observing the order of succession of the waves given by their velocities. The solution of the moving load problem is constructed in the same way as the solution of the sudden load problem at the boundary of a non-linear elastic half-space [6, 7]. The latter references give all possible solutions in the form of systems of quasi-transverse waves, depending on the position of the initial point σ_{i1}^* and the position of the final point σ_{i1}^γ.

The anisotropy of the state in front of the quasi-transverse wave system, on which the qualitative forms of the integral curves of the simple quasi-transverse waves and shock depend, and the polars of the quasi-transverse shock waves, are given by the quantity g [6, 7], which by assumption does not exceed ε. If the anisotropy is due to the initial deformation, then

$$g\approx\frac{\mu+3/4\gamma}{2\mu\sqrt{\mu\rho_0}}(\sigma_{22}-\sigma_{33})$$

where γ is the elastic constant of the medium. The difference $\sigma_{22}-\sigma_{33}$ depends on the initial deformation, on the intensity of the quasi-longitudinal wave (given by σ_{i1}^0 and σ_{i1}^γ), and on W. (This will be shown below.)

3. We construct a system of quasi-transverse waves for the case of a general situation when g is of order ε and the stresses in the region between the quasi-longitudinal wave and the system of quasi-transverse waves σ_{i1}^* is also of order ε.

The behaviour of quasi-transverse simple and shock waves greatly depends on the sign of the elastic constant κ [6, 7]. We will consider the case when $\kappa>0$. The integral curves of the simple quasi-transverse waves are two orthogonal families of straight lines for the fast and slow waves [1]. The fast simple waves correspond to ψ_1 in Eq. (2.2) $(\psi_1>\psi_2)$, and the integral curves of these waves are parallel to the σ_{21} axis (Fig. 1). The slow waves correspond to ψ_2

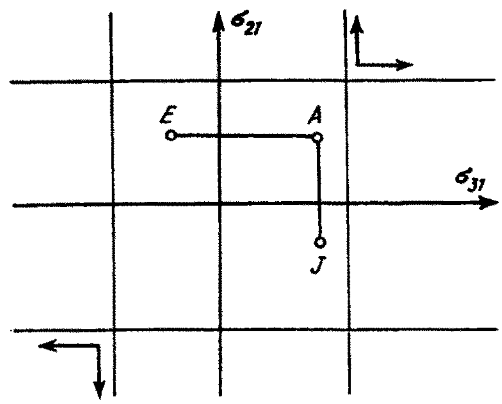

FIG. 1.

and their integral waves are parallel to the σ_{31} axis. The fast and slow simple waves do not reverse when the conditions $\psi_1>0$, $\psi_2>0$ are satisfied [1]. The arrows in Fig. 1 show the direction of change of the quantities σ_{31} and σ_{21} for quasi-transverse simple non-reversing waves.

The shock polar of quasi-transverse shock waves consists of two straight lines $\sigma_{21}=\sigma_{21}^*$, $\sigma_{31}=\sigma_{31}^*$ intersecting at the initial point A (σ_{31}^*, σ_{21}^*). The slow shock waves correspond to the evolution segment AE (Fig. 1) of the shock polar $\sigma_{31}^*\geqslant\sigma_{31}\geqslant-\frac{1}{2}\sigma_{31}^*$ along the line $\sigma_{31}=\sigma_{31}^*$ [1, 6]. Fast shock waves correspond to the evolution segment AJ (Fig. 1) of the shock polar $\sigma_{21}^*\geqslant\sigma_{21}\geqslant-\frac{1}{2}\sigma_{21}^*$ on the line $\sigma_{21}=\sigma_{21}^*$.

J and E are Jouguet points. At J the quantity ψ, calculated along the shock polar [1], is identical with the ψ_1 computed along the integral curve of the fast simple wave using (2.2). At E we similarly have $\psi=\psi_2$.

Figure 2 shows the systems of quasi-transverse simple and shock waves which are the solutions for various positions of the points σ_{21}^γ and σ_{31}^γ. The following notation is used in Fig. 2: R_1 and R_2 are, respectively, the slow and fast simple waves, S_1 and S_2 are the slow and fast shock waves, S_{1E} is the slow Jouguet shock wave ($A\to E$ jump), and S_{2J} is the fast Jouget shock wave (the jump $A\to J$).

The solution is a sequence of fast and slow waves. In regions 3, 10 and 6 the solution contains the fast compound wave $S_{2J}R_2$. A fast compound wave comprises two single-type waves moving with close velocities, and in this case they are a fast Jouguet shock wave and a fast simple wave. In regions 4, 5 and 6 the solution contains the slow compound wave $S_{1E}R_1$. A slow compound wave is a sequence of a slow Jouguet shock wave and a slow simple wave.

In slow waves with $\kappa>0$ the σ_{31} component varies, and for fast waves the σ_{21} component varies.

The solution of the problem for $\kappa<0$ is constructed similarly [7] and is also a sequence of two waves.

A quasi-longitudinal wave can change the sign of the difference $\sigma_{22}-\sigma_{33}$ without changing its order of magnitude. In this case the solution for the quasi-transverse wave system is also a sequence of two waves. However, it is now the σ_{31} component that varies in the fast wave and σ_{21} in the slow wave, i.e. the order of wave succession changes.

The quantity g, describing the anisotropy of the medium in the quasi-transverse wave, changes because of the deformation in the quasi-longitudinal wave, and cases are possible when g is reduced so that its magnitude is no greater than ε^2. In this case the effects of weak non-linearity and weak anisotropy become comparable with one another and the part of the solution for the moving load problem which corresponds to the quasi-transverse wave system has a more complicated picture [6, 7].

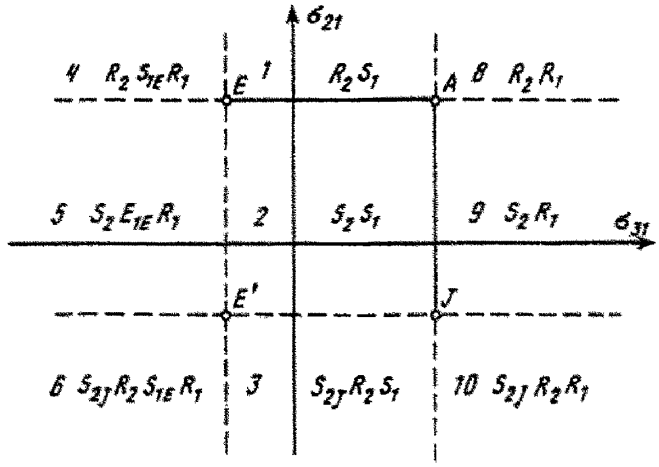

FIG. 2.

4. The investigation performed above shows that the global solution of the non-linear self-similar moving load problem is constructed as follows. In the space σ_{i1} $(i = 1, 2, 3)$ in which the points σ_{i1}^0 and σ_{i1}^γ are specified, the π plane with normal $\mathbf{n}$ $(\cos\alpha;\ -\sin\alpha;\ 0)$ passes through the point σ_{i1}^γ (Fig. 3). We then construct a line with direction vector $\mathbf{n}_1$ $(\cos\alpha_1;\ -\sin\alpha_1;\ 0)$ from the point σ_{i1}^0 up to its intersection with the π plane. This defines the point σ_{i1}^*.

Changing from coordinates σ_{21}', σ_{31}' in the π plane to σ_{i1} space coordinates, the orthogonal basis vectors (0; 1; 0) and (0; 0; 1) in the π plane become orthogonal vectors $(\sin\alpha;\ -\cos\alpha;\ 0)$ and (0; 0; 1) in the σ_{i1} space, i.e. the transition from investigating quasi-transverse wave solutions in the σ_{21}', σ_{31}' plane to the original σ_{i1} space preserves angles.

One must define the σ_{21}', σ_{31}' axes in the π plane. To this end we first define the point O_1 which is the point of intersection of the σ_{11} axis and the line l which is the intersection of the π plane with the σ_{11}, σ_{31} plane. The line l_1 is the intersection of the π plane with the σ_{11}, σ_{21} plane The lines l and l_1 define the coordinate axes σ_{21}' and σ_{31}' with direction vectors (0; 0; 1) and (sin α; cos α; 0). The origin of coordinates O is the point $(2q/\kappa,\ 2p/\kappa)$. The coordinate axes σ_{21}^n, σ_{31}^n are obtained by rotating the σ_{21}', σ_{31}' axes through an angle φ (2.1) (Fig. 3). We then fix the points $E(-\tfrac{1}{2}\sigma_{31}^*;\ \sigma_{21}^*)$, $J(\sigma_{31}^*;\ -\tfrac{1}{2}\sigma_{21}^*)$, $E'(-\tfrac{1}{2}\sigma_{21}^*;\ -\tfrac{1}{2}\sigma_{21}{}^*)$ in the π plane with σ_{21}^n, σ_{31}^n coordinates. The straight lines AJ, AE, EE' and $E'J$ divide the π plane into nine domains in which the solution is constructed by the method described above.

When the boundary conditions σ_{i1}^γ change the π plane changes its position in the σ_{i1} space whilst remaining orthogonal to the vector $\mathbf{n}$. The coordinates of the point A (σ_{i1}^*) therefore change. In the π plane the point A moves along a line that is the projection onto the π plane of the line passing through the point σ_{i1}^0 with direction vector $\mathbf{n}_1$. Because $\mathbf{n}_1$ lies in a plane orthogonal to the π plane, the point A obviously moves along the line l_1 in the π plane.

Furthermore, when σ_{i1}^γ changes, the position of the σ_{21}^n, σ_{31}^n axes in the π plane also changes because the intensity of the quasi-longitudinal wave occurring in the solution changes. As was noted previously, the passage of the quasi-longitudinal wave changes the quantity g^n characterizing the anisotropy of the medium. The angle φ governing the position of the axes σ_{21}^n, σ_{31}^n depends on g^n (2.1). Thus when σ_{i1}^γ changes the position of the lines AJ, AE, EE' and $E'J$ changes in the σ_{i1} space and the family of these lines are surfaces on which the type of solution changes.

The point A lies on the σ_{21}^n axis if the equation

$$\operatorname{tg}\varphi = \frac{\sigma_{31}^n - 2q/\kappa}{\sigma_{21}^n - 2p/\kappa} \tag{4.1}$$

which follows from formulae (2.1), is satisfied, and on the σ_{31}^n axis if the equation

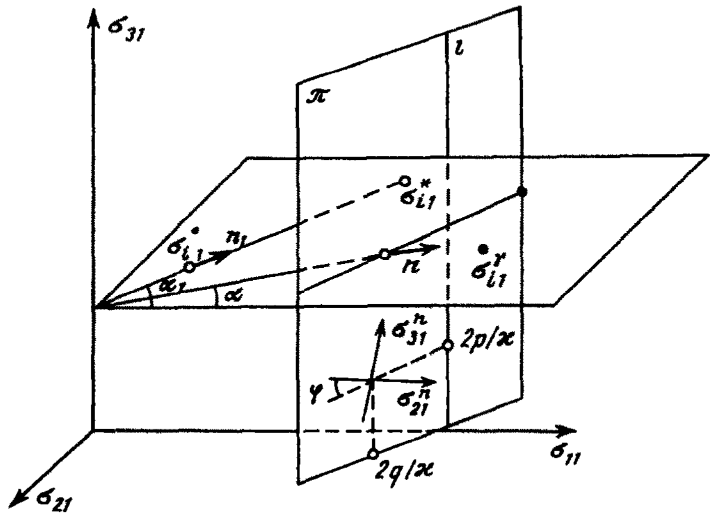

FIG. 3.

$$\operatorname{tg}\varphi=\frac{\sigma^n_{21}-2p/\kappa}{\sigma^n_{31}-2q/\kappa} \tag{4.2}$$

is satisfied.

If Eq. (4.1) is satisfied the point A lies on the σ^n_{21} axis and coincides with the point E. In this case the π plane is decomposed into three regions (regions 1–6 are not present), and the solution is symmetric about the σ^n_{21} axis. If Eq. (4.2) is satisfied, the point A lies on the σ^n_{31} axis and coincides with the point J. In this case the π plane is also decomposed into three regions (regions 2, 3, 5, 6, 9, 10 are not present), and the solution is symmetric about the σ^n_{31} axis.

The forms of the surfaces separating the different types of solution can be graphically illustrated in the case of an isotropic body with $\sigma^0_{\alpha} = 0$. The stressed state after the passage of the quasi-longitudinal wave in front of the transverse wave system is described by the following stress tensor components

$$\Delta\sigma'_{22}=\sin^2(\alpha_1-\alpha)\,\Delta\sigma''_{11},\quad \Delta\sigma'_{31}=0$$
$$\Delta\sigma'_{23}=\Delta\sigma'_{32}=0,\quad \Delta\sigma'_{33}=0$$

We shall assume that the quantity $\sigma'_{22}-\sigma'_{33}$ is of the order of ε or less. We will find the principal directions of the tensor $\sigma'_{\alpha\beta}$ (α, $\beta=2, 3$). They are determined by the eigenvalues of the matrix components of the tensor $\sigma'_{\alpha\beta}$: $\lambda_1=1$, $\lambda_2=0$. The principal axes therefore coincide with the lines l_1 and l (Fig. 3). The point A corresponding to the stressed state between the quasi-longitudinal wave that has

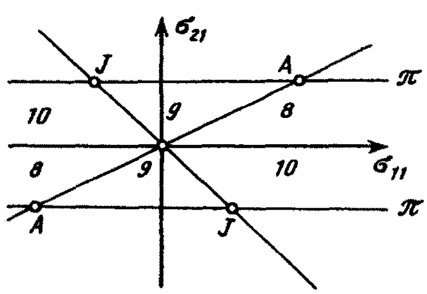

FIG. 4.

passed and the quasi-transverse system of waves lies on the line with direction vector $\mathbf{n}_1$ passing through the point σ_n^0. Consequently, for the case of an isotropic body the point A lies on the line l_1 and when the boundary conditions σ_n^γ change it moves in the plane π along this line. Consequently the point J, which in the case under consideration lies on the line l_1, is also displaced along this line.

Thus the surfaces dividing the various types of solution in the isotropic body case are three intersecting planes. Their mutual positions, projected onto the $\sigma_{11}\sigma_{21}$ plane, are depicted in Fig. 4. In regions 8 the solution is a succession of fast and slow simple waves, in regions 9 the solution is a succession of a fast shock wave and a slow simple wave, and in regions 10 is a compound fast wave and slow simple wave.

The solution of the self-similar problem of the effect of a moving load at the boundary of a non-linear elastic half-space is an essential part of the solution of the two-body collision problem.

I wish to thank A. G. Kulikovskii, Ye. I. Sveshnikova and A. A. Barmin for their comments and interest.

REFERENCES

1. CHUGAINOVA A. P., Stationary quasi-transverse simple and shock waves in a weakly-anisotropic elastic medium, *Prikl. Mat. Mekh.* **55**, 3, 486-492, 1991.
2. WRIGHT T. W., Reflection of oblique shock waves in elastic solids. *Int. J. Solids Struct.* **7**, 2, 161–181, 1971.
3. WRIGHT T. W., Oblique reflections. In *Propagation of Shock Waves in Solids*, pp. 79–95. New York, 1976.
4. BURENIN A. A., LAPYGIN V. V. and CHERNYSHOV A. D., The solution of plane self-similar problems of the dynamic non-linear theory of elasticity. In *Non-linear Deformation Waves* (Math. Symp.) Vol. 2, pp. 25–28. Tallinn Inst. Kibern., Akad. Nauk ESSR, 1978.
5. LAX P. D., Hyperbolic systems of conservation laws, II. *Communs Pure Appl. Math.* **10**, 4, 537–566, 1957.
6. KULIKOVSKII A. G. and SVESHNIKOVA Ye. I., The self-similar problem of the action of a sudden load on the boundary of an elastic half-space. *Prikl. Mat. Mekh.* **49**, 2, 284–291, 1985.
7. KULIKOVSKII A. G. and SVESHNIKOVA Ye. I., Non-linear waves produced by a stress change at the boundary of an elastic half-space. In *Problems of Non-linear Mechanics of a Continuous Medium*, pp. 133–145. Valgus, Tallinn, 1985.
8. KULIKOVSKII A. G. and SVESHNIKOVA Ye. I., Non-linear waves in weakly anisotropic elastic media. *Prikl. Mat. Mekh.* **52**, 1, 110–115, 1988.

Translated by R.L.Z.